Frequency characteristic

1. Plot the magnitude and phase characteristic of elements a,b, c, d and e.
   a. Harmonic signal should be connected to the input of the element. Observe input and output on the Scope. Note the amplitude value and time shift between maximum value in one period in input and output signal. (Ampl and Δt in Table 1). Calculate ϕ.
      Change frequency in range range 0.1 Hz – 100 Hz. Each step frequency should be multiply by 2, e.g. 0.1 Hz, 0.2 Hz, 0.4 Hz ….
   b. Plot results with logarithmic and linear scale (Fig.1 and Fig.2).
   c. Determine the resonant frequency f_r for system e (MDS mass-dissipative-spring). Connect the step signal to the input and observe output. Data: m = 10 kg, k = 20 N/cm (!), h = 50 N/(m/s). Recalculate units to SI system (kg, m, m/s, N ...)
   d. Determine amplitude and phase characteristic in analytic way for systems b, c and d. Plot the frequency characteristics for each system. Compare results from simulation with analytic solution.

2. In system e determine the critical value of damping factor (modify h parameter to obtain aperiodic response). Note value of *h* parameter and for that critical value calculate the nondimensional damping factor *D*.

3. Determine individual parameters of the system depend on your name and student ID (e.g. Jan Nowak, ID 50789)

   k=length of the first name (3)
   T=length of the last name (5)
   a=fifth digit of the ID (9)
   b= fourth digit of the ID (8)

a. Proportional G(s)=k

b. Integrator $G(s) = \frac{1}{Ts}$

c. Derivative $G(s) = Ts$

d. First order system $G(s) = \frac{1}{as + b}$

Differential equation: $\boxed{a\dot{y} + by = x}$

Operator equation: *a s y(s) + b y(s) = x(s)*

hence $$\frac{y(s)}{u(s)} = \frac{1}{as+b} = \frac{\frac{1}{b}}{Ts+1}$$ where *T = b/a;*

step response *x(t)* = c***1**(*t*)

$$\boxed{y(t) = y_0 + c \cdot \left(1 - e^{-\frac{t}{T}}\right)}$$

where y_0 – initial value *y*, *y(t=0)*.

Value of the signal y in given time (after k times constants) *t = k*T*:

$$\boxed{y(t) = y_0 + c \cdot \left(1 - e^{-\frac{kT}{T}}\right) = y_0 + c \cdot (1 - e^{-k})}$$

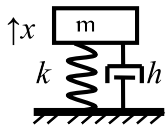
For k=1 ($t = T$):

$$\boxed{\frac{y(T)}{c} = \frac{y_0}{c} + (1 - e^{-1}) \approx \frac{y_0}{c} + 0{,}63212}$$

For $y_0 = 0$ $\qquad y(T) \approx 0{,}63212 \quad c$

e. Second order system (oscillator) $G(s) = \dfrac{c}{as^2 + bs + 1}$

Forces in the vertical direction: $\downarrow F$



1. Inertia force: $F_b = m\ddot{x}$
2. Spring force: $F_k = k\,x$
3. Dumping force: $F_h = h\dot{x}$
4. External force F

$m\ddot{x} + h\dot{x} + kx = F$ Laplace Transform: $(ms^2 + hs + k)\,x(s) = F(s)$

Transmittance e(jω) = x(j ω) /F(j ω) is susceptibility.

$$e(j\omega) = \frac{1}{m\cdot(j\omega)^2 + h\cdot(j\omega) + k} = \frac{\frac{1}{k}}{\frac{m}{k}(j\omega)^2 + \frac{h}{k}(j\omega) + 1}$$

With : $\dfrac{k}{m} = \omega_0^2;\quad \dfrac{h}{k} = \dfrac{h}{m\omega_0^2} = \dfrac{h}{m\omega_0}\cdot\dfrac{1}{\omega_0} = 2D\dfrac{1}{\omega_0}$ following results is obtained:

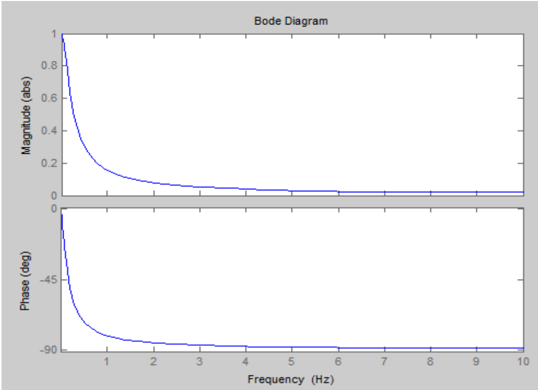
$$e(j\omega) = \frac{\frac{1}{k}}{\frac{1}{\omega_0^2}(j\omega)^2 + \frac{2D}{\omega_0}(j\omega) + 1}$$

where ω_0 natural frequency and D nondimensional damping factor

Transmittance: $e(s) = \dfrac{c}{as^2 + bs + 1}$

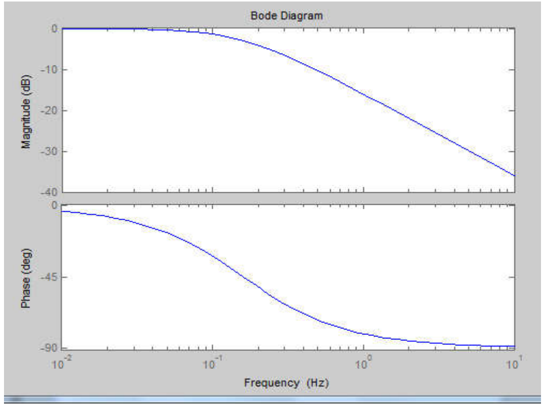
Table 1. (eg. Element a)

f [Hz]	Δt [s]	Ampl []	ϕ = Δt*360*f [deg]
0.1			
0.2			
0.4			
…			
10	(2.375-2.4)	1.25	(2.375-2.4)*360*10=-90
…			
100			



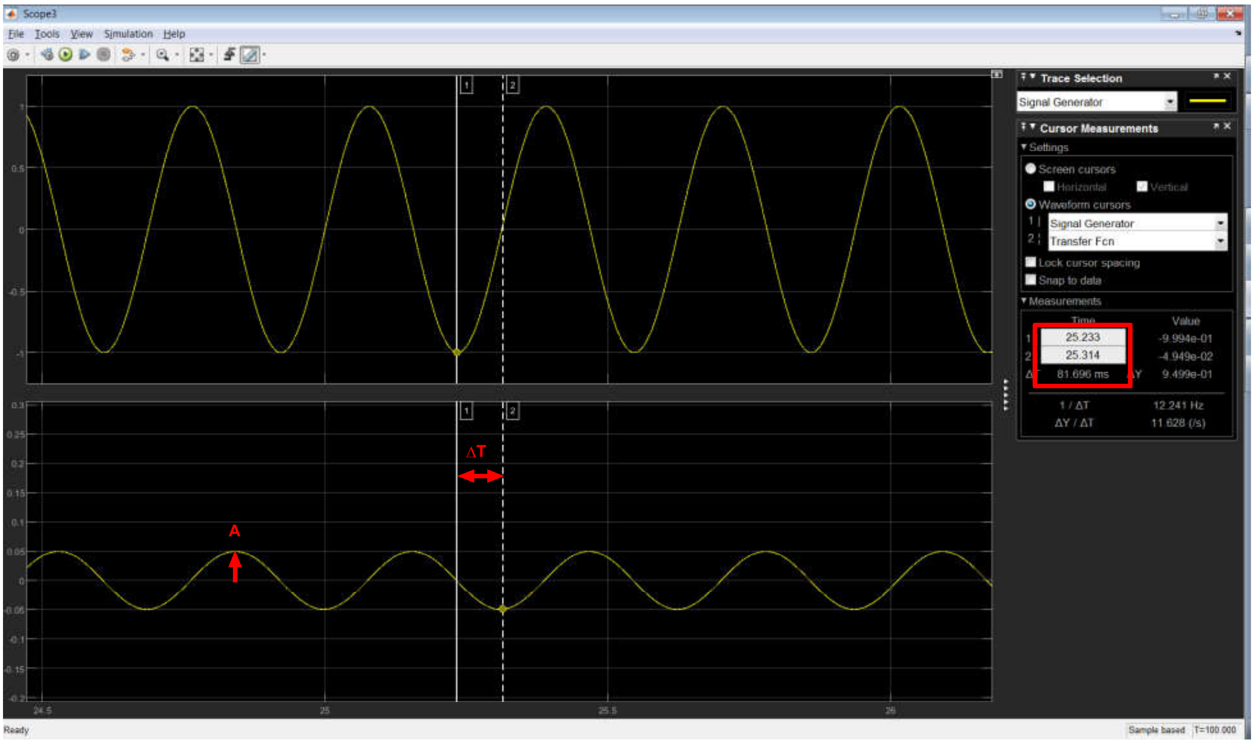
Bode Diagram
Magnitude (abs)
Phase (deg)
Frequency (Hz)

Fig. 1



Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (Hz)

Fig. 2



Scope3
File Tools View Simulation Help
Trace Selection
Signal Generator
Cursor Measurements
Settings
Screen cursors
Horizontal
Vertical
Waveform cursors
Signal Generator
Transfer Fcn
Lock cursor spacing
Snap to data
Measurements
Time
Value
25.233
-9.994e-01
25.314
-4.949e-02
81.696 ms
9.499e-01
1 / ΔT
12.241 Hz
ΔY / ΔT
11.628 (/s)
ΔT
A
Ready
Sample based
T=100.000