

Lead / Lag Compensator

1. Scope

Examine properties of the linear dynamic system compensated by Lead / Lag Compensator.

2. Control object

Define transfer function of the plant e.g.

```
G=tf([50],[0.2 1 0])
```

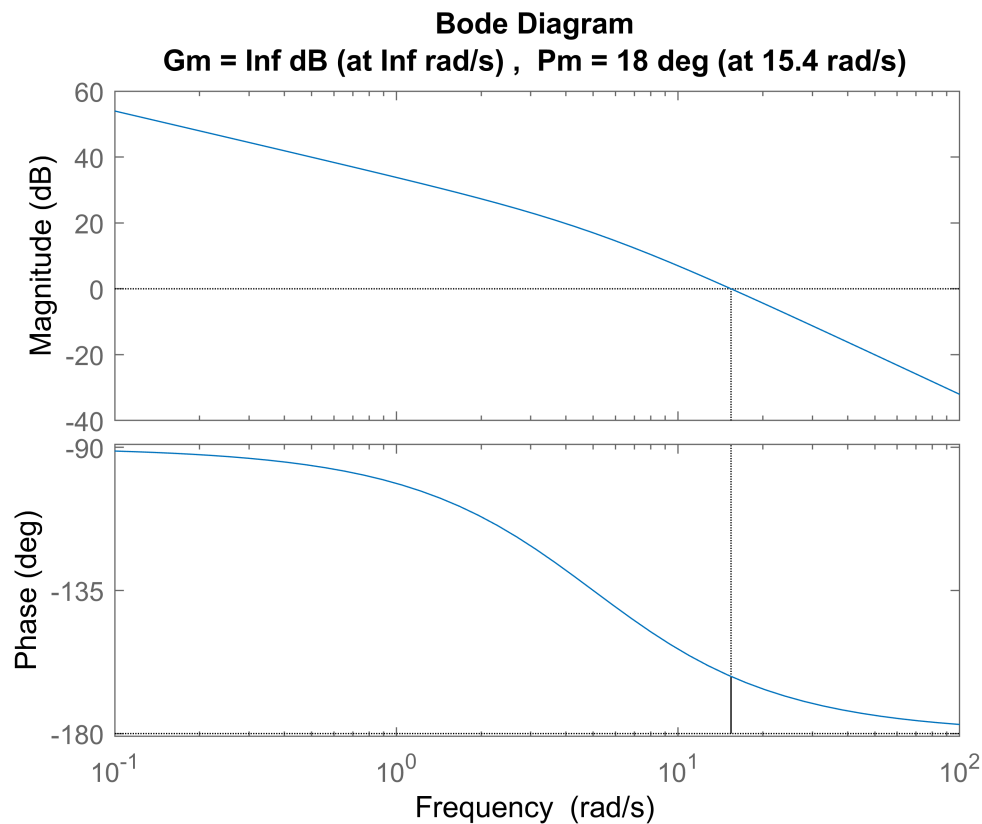
G =

$$\frac{50}{0.2 s^2 + s}$$

Continuous-time transfer function.

Check dynamic properties of the plant by examining the frequency characteristic and stability margin:

```
margin(G)
```



You can find, that phase margin PM is only 18 deg. In order to modify the system properties Lead or Lag Compensator could be used.

3. Lead Compensator

Define compensator in the form:

```
a=2; tau=0.05;  
Lead=tf([a*tau 1],[tau 1])
```

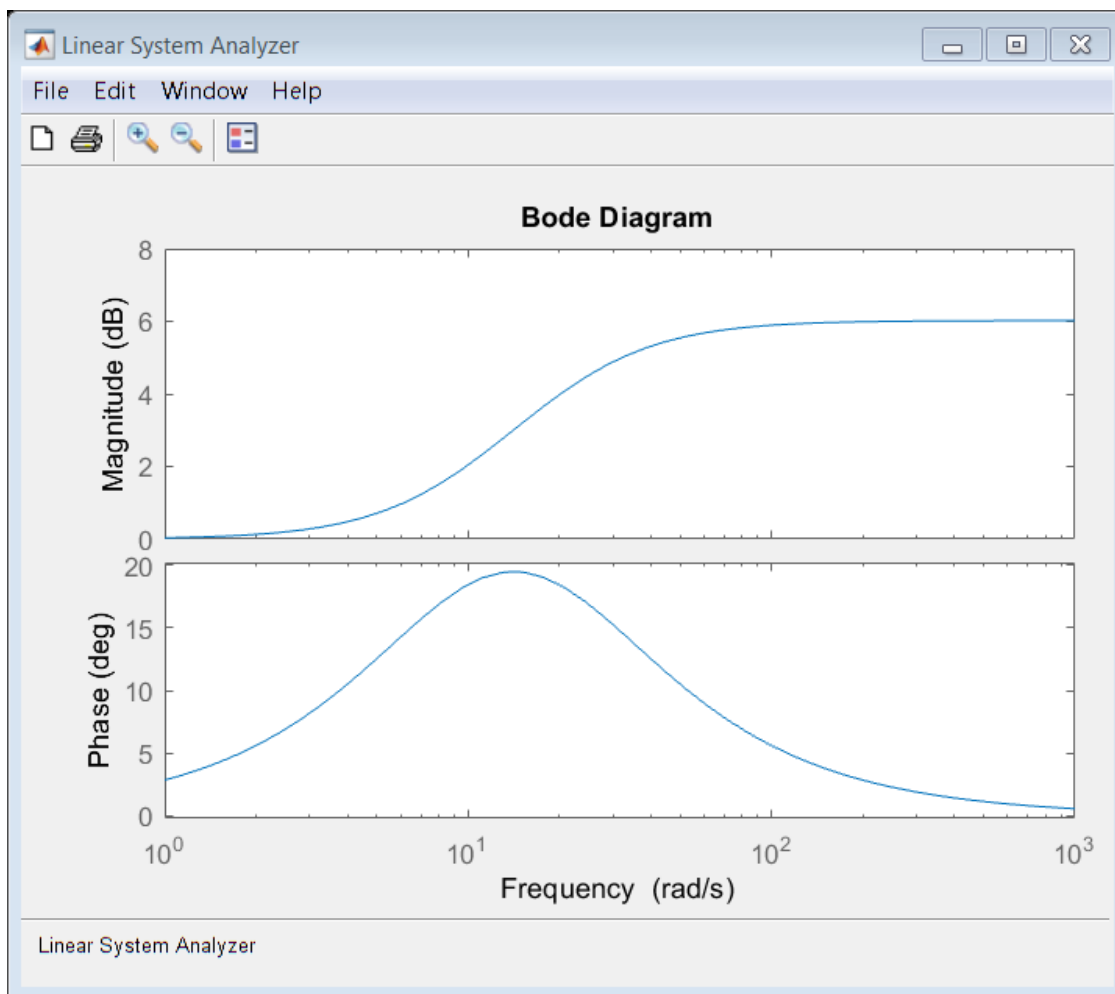
Lead =

$$\frac{0.1 s + 1}{0.05 s + 1}$$

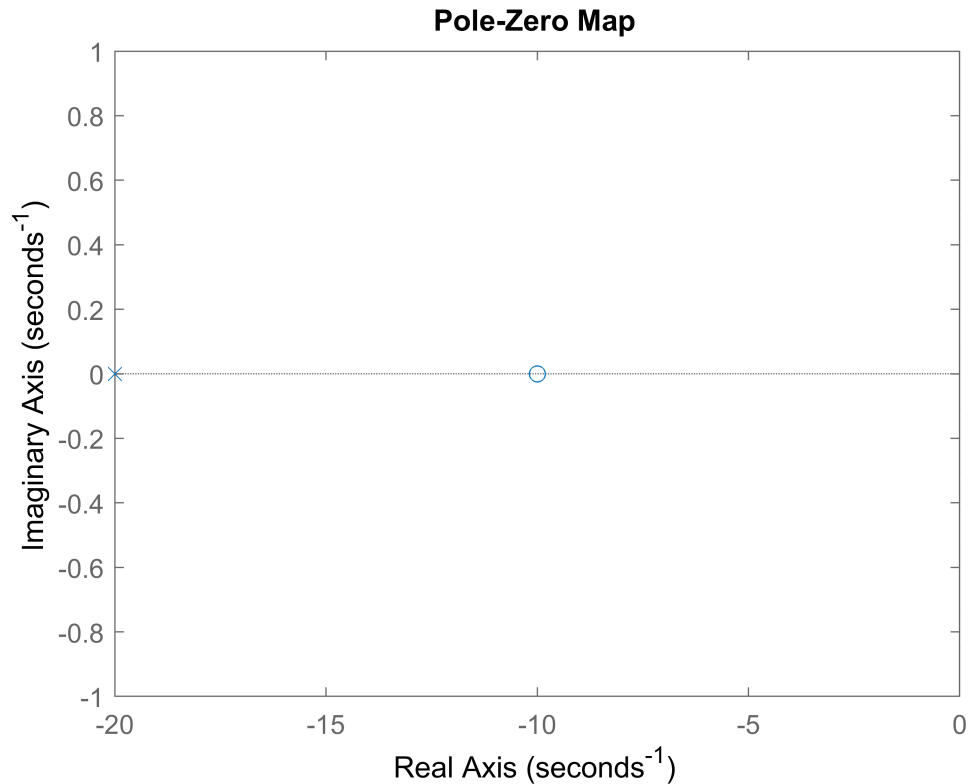
Continuous-time transfer function.

Examine properties of the compensator

```
ltiview('bode',Lead);
```



```
pzmap(Lead);
```



In the Lead Compensator the pole should be always located on the left side of zero. Change the parameters of compensator (a and tau) and present results of bode diagram for each case. Parameter a should be always greater than 1.

To increase phase margin for e.g. 50 degree the compensator should be able to shift phase characteristic about 30 degree (fi_max) at 15 rad/sec (w_max). On this basis calculate the a and tau according to following equations:

```
a=4;tau=0.03;
fi_max=asin((a-1)/(a+1))*180/pi
```

fi_max = 36.8699

```
w_max=1/(tau*sqrt(a))
```

w_max = 16.6667

Define Lead compensator with the new parameters:

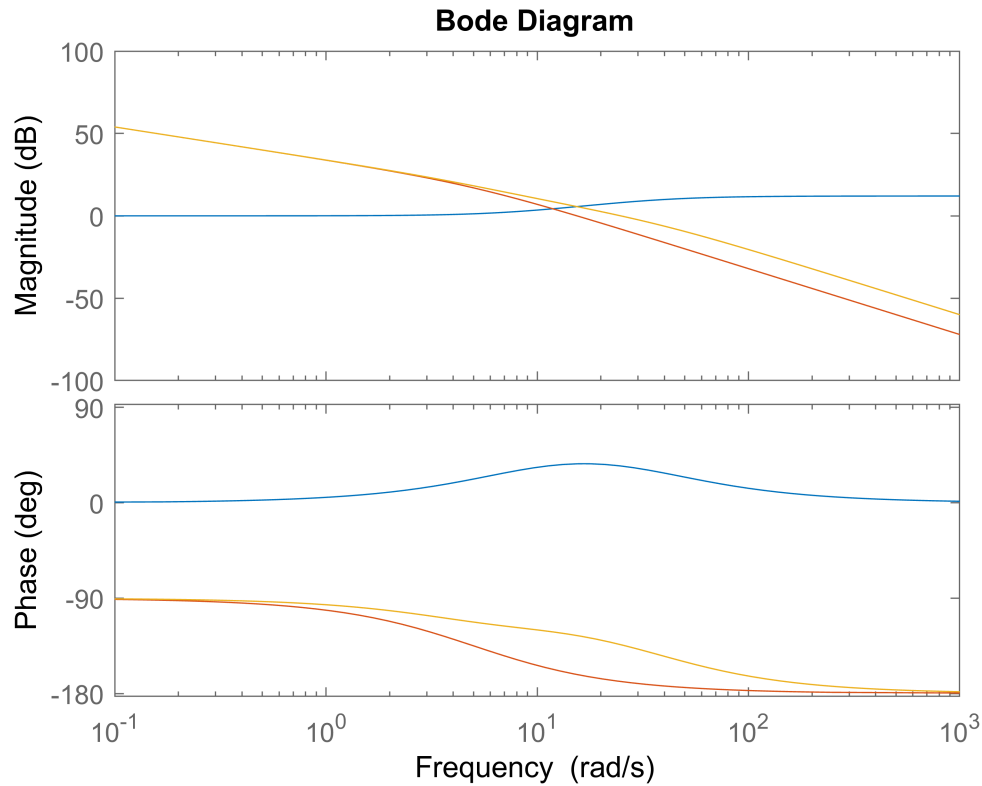
```
Lead=tf([a*tau 1],[tau 1])
```

```
Lead =
      0.12 s + 1
      -----
      0.03 s + 1
```

Continuous-time transfer function.

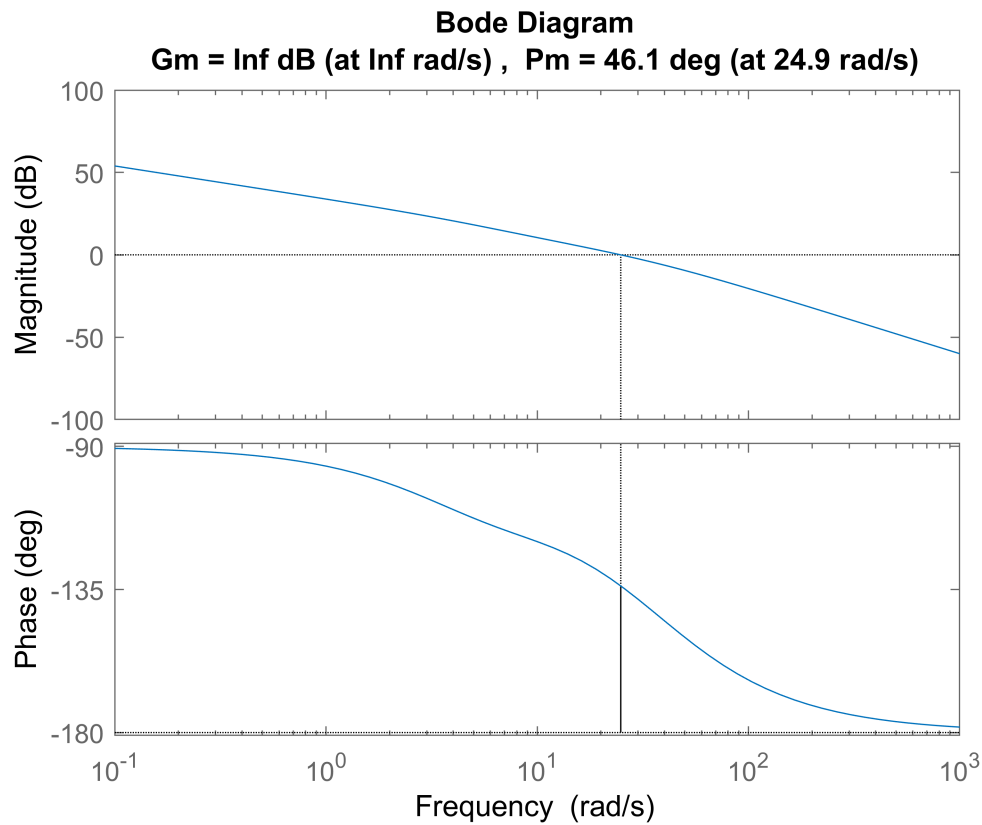
Present bode diagrams for Lead, Plant G and compensated plant (two systems connected) (G*Lead).

```
bode(Lead,G,Lead*G)
```



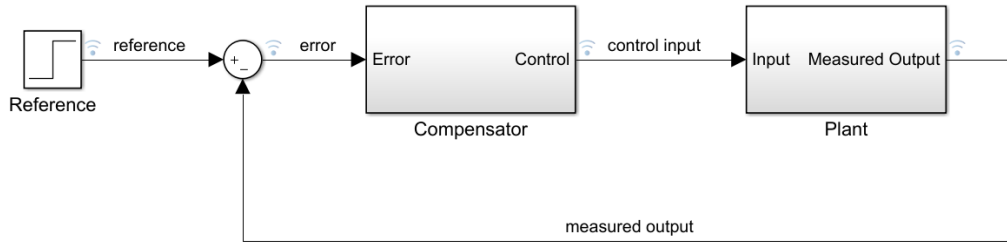
Check what happen with characteristics? Determine the phase margin for plant and compensated plant? Is PM=50 degree? If not, try to explain why.

```
margin(Lead*G)
```



Check the step response of plant and compensated plant and comment results

4. Build simulation according to following diagram:



Check the close loop step response for system for compensated and uncompensated plant. Comment results in view of output, control and error signal.

Instead of Lead compensator use PID controller. Try to tune PID controller and compare results.

[1] DISTEFANO J.J., STUBBERUD A.R., WILLIAMS W.J.: Control and Feedback Systems.

[2] Brian Douglas: Why learn control theory. YouTube video.