PID Control System

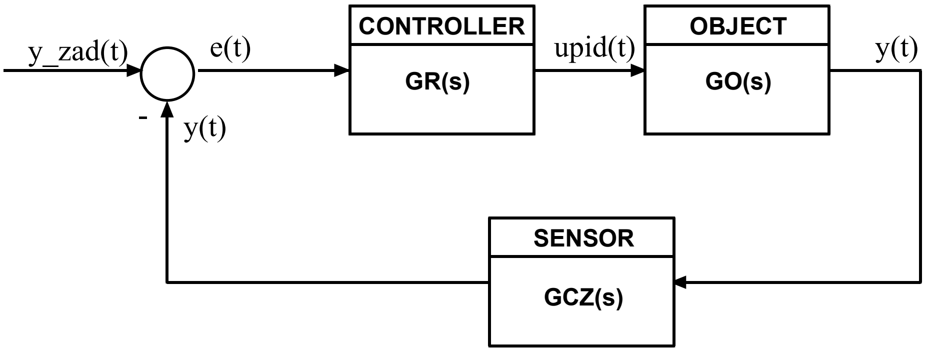
1. Scope

Frequency characteristic of the P, PI, PD, PID elements. Tuning PID controller based on the Ziegler-Nichols principle. Tuning PID controller based on Kupfmuller model.

Parameters of the systems are individual tuned for each Student. Determine individual parameters of the system depend on your name and student ID (e.g. Jan Nowak, ID 50789)

Kp=length of the first name (3)
Ti=length of the last name (5)
Td=fifth digit of the ID (9)
b= fourth digit of the ID (8)

2. Closed loop control diagram



PID controller structure is composed from following elements:

P: Proportional: up(t)=Kp*e(t); Gp(s)=Kp,

I: Integral: $uc(t)=\frac{1}{Ti}\int_0^T e(t)dt, \qquad G_I=\frac{1}{Ti*s}$

D: Derivative: $ur(t)=Td\frac{d}{dt}e(t), \qquad G_R=Td*s$

Classic PID structure:

$$upid(t)=Kp*e(t)+Kp\frac{1}{Ti}\int_0^T e(t)dt+Kp*Td\frac{d}{dt}e(t), \qquad G_{PID}=Kp(1+\frac{1}{Ti*s}+Td*s)$$

The ideal derivative element $G_R=Tr*s$ cannot be realized from technical point of view. The real derivative element is described by $G_R=\frac{Td*s}{Tdi*s+1}$, $Tdi<<Td$ and consist of integrator.

In results the real PID controller is given by $G_{PID}=Kp\left(1+\frac{1}{Ti*s}+\frac{Td*s}{Tdi*s+1}\right)$.

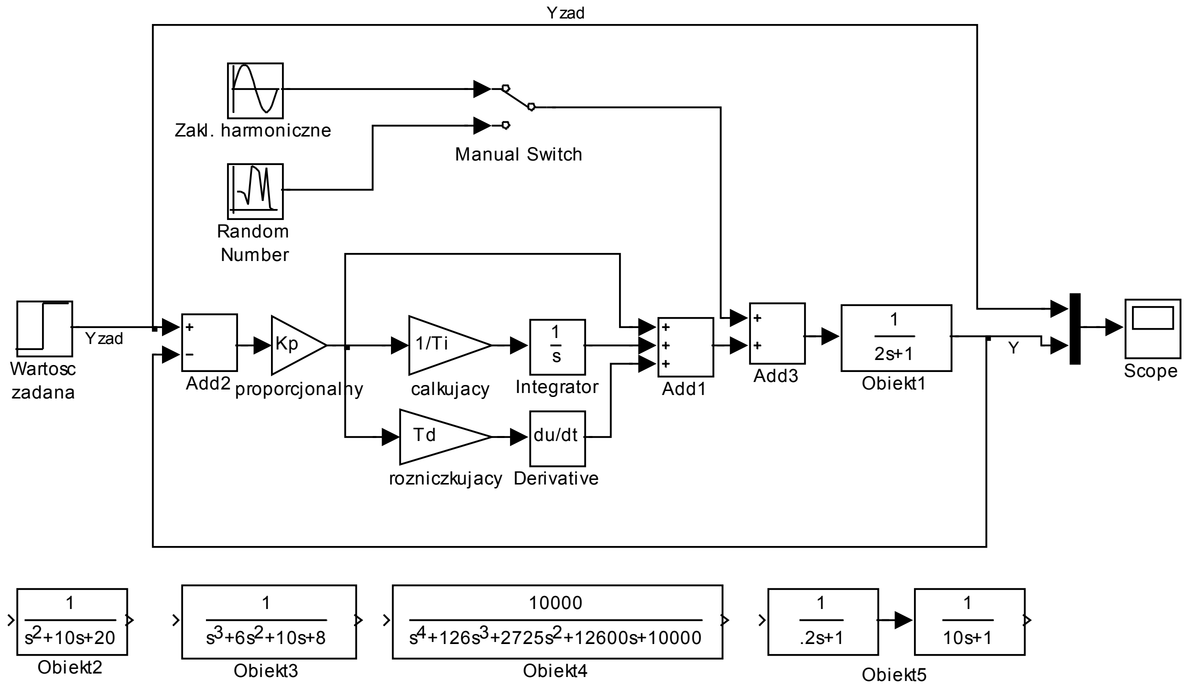
Plot the bode and Nyquist characteristics of all elements:

1. P, 2. I, 3. D, 4. PI, 5.PD, 6.PID

Define transfer function G in workspace and use instruction: *bode(G)* i *nyquist(G)*. Parameters kp, Ti, and Td should be determine according to the point 1.

3. Tuning PID controller in experiment

Prepare diagram in Simulink:



Tune Kp, Ti i Td for Obiekt1 and Obiekt2 in view of time response, steady state error end overshot Note in the table influence of P, I, D elements on the response time, steady state error and overshoot. E.g. increasing Kp results in shortening the response time, etc. **Perform simulation without disturbance.**

	Response time [s]	Steady state error	Overshot [%]
Kp			
Ti			
Td			

Find optimal (in your opinion) value of **Kp, Ti i Td.** Tune controller in following sequence:

1. Only P element (I, D - disabled),
2. Enable D and retune P,
3. Enable I and retune P, D.

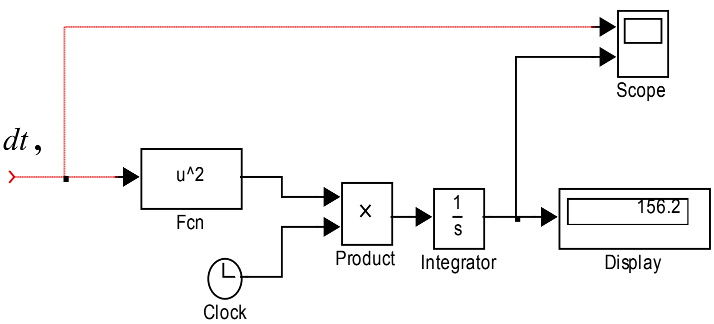
4. Quality of control system

Prepare diagram in Simulink. Copy and modify it to achieve all indexes.

Integral Absolute Error $IAE = \int_0^T |e(t)|\, dt$,

Integral Time Absolute Error $ITAE = \int_0^T t|e(t)|\, dt$,

Integral Square Error $ISE = \int_0^T e^2(t) dt$,



Integral Time Square Error $ITSE = \int_0^T te^2(t)dt$,

To the red line should be connected to the error signal e(t). The error signal could be located according to the diagram in p. 2. To enable second input in scope go to PARAMETERS and in NUMBER OF AXES = 2.

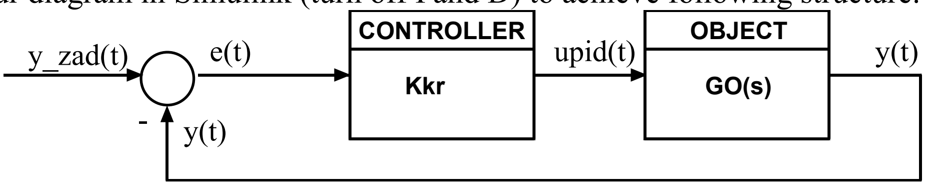
Execute simulation without disturbance for Obiekt2 with step input signal for P, PI and PID controller. Determine value of the ITSE and ISE, overshot and time response:

- P: Kp=300,
- PD: Kp=300, Td=1/30,
- PI: Kp=30, Ti=3/7,
- PID: Kp=300, Ti=1.2, Td=0.075.

Enable disturbance and perform simulation for sine and random disturbance signal. Observe and save output y(t), disturbance and ISE, ITSE.

5. Tuning PID based on the Zigler-Nichols principle

Modify your diagram in Simulink (turn off I and D) to achieve following structure:



Increase the Kkr and observe output. Note value Kkr when non damping oscillation occurs in the output signal. Determine the oscillation period Tosc. This methods gives the Best results when: $2 < K * Kkr < 20$

Determine value of the PID parameters:

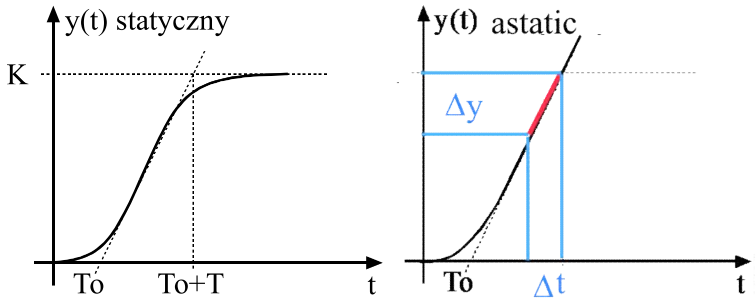
Controller type	Kp	Ti	Td
P	0.5Kkr	-	-
PI	0.45Kkr	$\frac{Tosc}{1.2}$	-
PID	0.6Kkr	$\frac{Tosc}{2}$	$\frac{Tosc}{8}$

Tune P, PI, PID controller for Obiekt3 and next Obiekt2 (GO(s)). Execute simulation with and without disturbance. Note value of ISE, ITSE and plot output signal.

6. Tuning PID based on the step response

Step response function can be approximated by:

$G(s) = \frac{Ke^{-sTo}}{Ts+1}$ for static and $G(s) = \frac{Ke^{-sTo}}{s}$ for astatic systems:



PID for static system with time delay $a = K\frac{To}{T}$

Controller type	Overshot 0%			Overshot 20%			Minimum ISE		
	Kp	Ti	Td	Kp	Ti	Td	Kp	Ti	Td
P	$\frac{0.3}{a}$	-	-	$\frac{0.7}{a}$	-	-	-	-	-
PI	$\frac{0.6}{a}$	$0.8To + 0.5T$	-	$\frac{0.7}{a}$	$To + 0.3T$	-	$\frac{1.0}{a}$	$To + 0.35T$	-
PID	$\frac{0.95}{a}$	$2.4To$	$0.4To$	$\frac{1.2}{a}$	$2.0To$	$0.4To$	$\frac{1.4}{a}$	$1.3To$	$0.5To$

PID for astatic system with time delay K= Δt/Δy

Controller type	Overshot 0%			Overshot 20%			Minimum ISE		
	Kp	Ti	Td	Kp	Ti	Td	Kp	Ti	Td
P	$\frac{0.37T}{To}$	-	-	$\frac{0.7T}{To}$	-	-	-	-	-
PI	$\frac{0.46T}{To}$	$5.75To$	-	$\frac{0.7T}{To}$	$3To$	-	$\frac{1T}{To}$	$4.3To$	-
PID	$\frac{0.65T}{To}$	$5To$	$0.23To$	$\frac{1.1T}{To}$	$2To$	$0.37To$	$\frac{1.36T}{To}$	$1.6To$	$0.5To$

Simulate step response for Obiekt3, 4 i 5. On this basis determine the parameters of the static or astatic models. Set P, PI, PID parameters from the table. Perform simulation for each case and determine step response, overshot, steady state error, ISE and ITSE with and without disturbance.

7. Report

- Diagrams, parameters, bode and Nyquist characteristic of PID controllers and objects (p.2)
- Time plots of the reference, output and error signal. (p.3 - 4)
- Summary

8. References